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Numerical Study of the Dynamics of a Cyclone Modeled by a Vertical Vorticity Vector Considering a Radial Velocity as a Function of Time

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Abstract: In this article, the work carried out is a numerical study of the dynamics of a cyclone modeled by a vertical vorticity vector by considering a radial velocity as a function of time t. We made a short general study on cyclones by modeling a vertical cyclone starting from the Navier-Stokes equations. A study of the vorticity as a function of the radius and as a function of the radial abscissa at different times has been made. We also studied the radial and ortho radial components of the velocity as a function of the radius and as a function of the radial abscissa by varying the Ekman number and the Rossby number by looking at their impacts when they are increased or they are reduced. In the same dynamics, the influence of the decrease in the suction speed over time on the radius of the vortex core has been studied. In short, we have studied the effects of the variation of the radial velocity as a function of time. We ended our study by showing that the Ekman number is the most important parameter in the dynamics of the cyclone, by showing the relationship that exists between the Ekman number and the growth of the rotational motion of a cyclone.

Keywords: Cyclone, Vorticity Vector, Navier-Stokes

1. Introduction

Nowadays the cyclone is one of the most impressive and devastating natural phenomena. Cyclonic systems generate strong vortex structures that are of interest to meteorology and the fluid dynamics of geophysical flows. In the tropical and subtropical regions of the continents, they are the source of a large part of the gusts of wind with very high speeds, storms and torrential rains observed, frequently associated with material damage and loss of human life.

The frequent observation of episodes of tropical cyclones raises the question of their probability of occurrence, generally assessed from the analysis of observations from geostationary satellites in visible and infrared light above the globe and forecasts. Knowing that cyclones are not punctual, but affect most of the tropical regions of the globe, it is necessary to analyze them in spatial and temporal dimensions, therefore from a dynamic point of view. [1, 2]

In this context, many studies have been carried out on geophysical and oceanic flows. Models have been developed with the aim of predicting the trajectories of cyclones in order to minimize the material damage caused.

Certainly investigations on the dynamics of three-dimensional vortices have been made, but to our knowledge no study on the explicit influence of the radial velocity has been carried out. However, in the case of a vertical vortex, this radial velocity plays the role of suction and therefore it is useful to know how it acts on the dynamics of the vortex. If the suction phenomenon increases or decreases, what is the behavior of the vortex?

The devastating power of a cyclone is exerted in three areas, the wind, the rain, the sea. [3-6]

2. Modeling a Cyclone

The objective of our study will be here to give a physical approach to what a cyclone is through theoretical modeling and then numerical modeling. Let us begin first of all to introduce an essential quantity in the study of the flows of fluids in rotation which is the vorticity.

1) The vorticity

The study of the deformations of a fluid element reveals a term of local rotation which is the antisymmetric part of the tensor of the velocity gradients. Thus vorticity will be a privileged tool to describe the local rotation movements inside a fluid.

2) The vorticity vector

The vorticity vector at a point r in space is defined by:

$\vec{\omega}(\mathrm{r}) = \overrightarrow{\mathrm{rot}}\vec{V}(\mathrm{r})$, where $\vec{V}(\mathrm{r})$ is the velocity field of the flow.

Vorticity occurs whenever the flow is not potential and therefore in viscous fluids.

In some cases the flow is potential outside a line (the "core") of small diameter compared to the overall size of the flow. The rotation of the fluid takes place around the "heart" where the vorticity is located. Such flows are called vortices and are found in atmospheric waterspouts, cyclones and hurricanes clearly visible over several hundred kilometres. This allows us to use the dynamic study of vorticity to materialize the deadly vortices that are cyclones. [7]

3) Vorticity transport equation

Consider the Navier-Stokes equation

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V}.\vec{\nabla}).\vec{V} = -\frac{1}{\rho}\vec{\nabla}P + \nu\Delta\vec{V} + \vec{g} \quad (1)$$

With

$\vec{V}$ he velocity field

P is the pressure field,

ρ the density of the fluid

ν its kinematic viscosity and

$\vec{g}$ represents the gravity field.

We will now establish the vorticity transport equation. If we assume that the density is constant then by applying the rotational operator to both sides of equation (1), it comes after calculation

$$\frac{\partial \vec{\omega}}{\partial t} + (\vec{V}\vec{\nabla})\vec{\omega} = (\vec{\omega}\vec{\nabla})\vec{V} + \nu\Delta\vec{\omega} \quad (2)$$

$\frac{\partial \vec{\omega}}{\partial t}$: describes the temporal evolution of the vorticity which is very important in unsteady state.

$(\vec{V}\vec{\nabla})\vec{\omega}$: characterizes the convective effect, inertial therefore of vorticity transport, is very important, especially in geophysical flow.

$\nu\Delta\vec{\omega}$: represents the effects of damping, ordering of the flow, therefore the viscous effects.

$(\vec{\omega}\vec{\nabla})\vec{V}$ which involves the variations of the speed in the direction of the vector vorticity since it contains the projection of the vector gradient on the direction of $\vec{\omega}$. It is this term that explains the phenomenon of stretching. It is an important mechanism of vorticity amplification and there fore of vortex intensification. [7, 8]

4) Stretching phenomenon

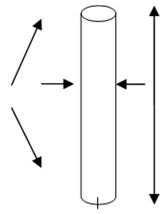


Figure 1. *Stretching of a vorticity tube.*

2.1. Theoretical Modeling of a Cyclone

Thus for the modeling we consider a uniform vorticity inside a cylindrical tube of radius r_0, infinitely elongated in the vertical direction in order to materialize the "eye" of the cyclone by the diagram below.

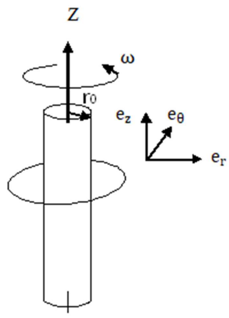


Figure 2. *Schéma représentant l'œil d'un cyclone en rotation.*

In this model, the vorticity is assumed to be purely axial at any time t, $\vec{\omega} = \omega\vec{e}_z$ and we will use the cylindrical coordinates (r, θ, z). [9]

2.2. The Navier-Stokes Equations

Consider the vorticity equation for an incompressible fluid flow.

$$\frac{\partial \vec{\omega}}{\partial t} + (\vec{V}\vec{\nabla})\vec{\omega} = (\vec{\omega}\vec{\nabla})\vec{V} + \nu\Delta\vec{\omega} \quad (3)$$

$$\mathrm{div}\vec{V} = 0 \quad (4)$$

To solve our problem, we make the following simplifying assumptions:

Symmetry axis assumptions:

For any scalar field F, we have

$$\frac{\partial F}{\partial \theta} = 0 \quad (5)$$

Hydrodynamic assumptions:

We assume that the velocity fields are finite at $r = 0$ at $z = 0$.

La vorticité est axiale c'est-à-dire $\vec{\omega} = \omega\vec{e}_z$

2.3. Determination of the ϑ_r and ϑ_z Components of the Velocity Field

Let us first start to study the continuity equation which will allow us to have the profile of the velocity field.

Taking into account the symmetry axis hypothesis, the continuity equation is reduced to

$$\mathrm{div}\vec{V} = \frac{\partial}{\partial r}(r\vartheta_r) + \frac{\partial}{\partial z}(r\vartheta_z) = 0 \quad (6)$$

Since the vorticity vector is unidirectional and with respect to the axis symmetry to the flow, we arrive at the following equalities:

$$\frac{\partial\vartheta_r}{\partial z} - \frac{\partial\vartheta_z}{\partial r} = 0 \quad (7)$$

$$\frac{\partial\vartheta_\theta}{\partial z} = 0$$

$$\vec{\omega} = \frac{\partial\vartheta_\theta}{\partial r}\vec{e}_z$$

From the last equation we see that the vorticity is a function only of the variable r.

$$\vec{\omega} = \omega(r,t)\vec{e}_z$$

He then comes

$$\frac{\partial\omega}{\partial z} = 0 \Rightarrow \frac{\partial^2\omega}{\partial z^2} = 0$$

On the one hand, as the partial derivatives with respect to the variables θ and z of the component of the following velocity vector $\vec{e}_\theta$ are identically zero, we deduce that the latter is a function only of the distance from the point M considered to the axis of rotation.

$$\frac{\partial\vartheta_\theta}{\partial z} = \frac{\partial\vartheta_\theta}{\partial\theta} \equiv 0 \Rightarrow \vartheta_\theta = \vartheta_\theta(r)$$

To have the profile of the components ϑ_r et ϑ_z velocity field we use the equalities

$$\frac{\partial}{\partial r}\frac{\partial\vartheta_r}{\partial z} = \frac{\partial^2\vartheta_z}{\partial r^2} \quad (8)$$

By permuting the order of derivation with respect to ret z in the expression above we obtain

$$\frac{\partial}{\partial z}\frac{\partial\vartheta_r}{\partial r} = \frac{\partial^2\vartheta_z}{\partial r^2} \quad (9)$$

$$\frac{1}{r}\frac{\partial\vartheta_z}{\partial r} + \frac{\partial^2\vartheta_z}{\partial r^2} + \frac{\partial^2\vartheta_z}{\partial z^2} = 0 \quad (10)$$

$$\Leftrightarrow \left(\frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2} + \frac{\partial^2}{\partial z^2}\right)\vartheta_z = 0 \quad (11)$$

So the Laplacian of the component of the velocity field along the z axis is zero.

$$\vartheta_z = \vartheta_z(z) = 2.az + b \quad (12)$$

$$\vartheta_r = \vartheta_r(r) = -a.r + \frac{k}{r}$$

With k an integration constant.

To determine the integration constants, we must return to the hydrodynamic boundary conditions.

To $z = 0$; $\vartheta_z = 0 \Rightarrow a \times 0 + b = 0 \Leftrightarrow b = 0$

so

$$\vartheta_z = \vartheta_z(z) = 2.az$$

Taking into account the hydrodynamic hypothesis in the vicinity of the "eye" (r = 0) we must have an integration constant equal to zero for our problem to make sense. So k = 0 finally the components of the velocity field following r and z are respectively:

$$\vartheta_r = -a.r \quad (13)$$

$$\vartheta_z = 2.a.z \quad (14)$$

The parameter a has the dimension of a frequency must be positive at all times so that we do not find ourselves in a situation where the current becomes downward.

We ask $a = a_0 f(t)$ with a_0 a positive result.

Taking into account simplifying assumptions and boundary conditions.

Let us return to the vorticity transport equation.

$$\frac{\partial\vec{\omega}}{\partial t} + (\vec{V}\vec{\nabla})\vec{\omega} = (\vec{\omega}\vec{\nabla})\vec{V} + \nu\,\Delta\,\vec{\omega}$$

Since the vorticity is axial and only depends on the variable r, we then obtain the following reduced equation:

$$\frac{\partial\omega}{\partial t} + \vartheta_r\frac{\partial\omega}{\partial r} = \omega\frac{\partial\vartheta_z}{\partial z} + \nu\left(\frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2}\right)\omega \quad (15)$$

Then by replacing

ϑ_r and ϑ_z formula speakers we arrive at the following expression.

$$\frac{\partial\omega}{\partial t} - a\,r\frac{\partial\omega}{\partial r} = 2a\omega + \frac{\nu}{r}.\frac{\partial}{\partial r}\left(r\frac{\partial\omega}{\partial r}\right) \quad (16)$$

which can be put in the following conservative form.

$$\frac{\partial\omega}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[a\,r^2 + \nu\,r\frac{\partial\omega}{\partial r}\right] \quad (17)$$

This is the vorticity evolution equation for the model studied here. It is this equation that we are going to comment on and adimensionalize in order to reveal dimensionless quantities that serve to characterize the dynamics of our "cyclone".

2.4. Initial Conditions on the Vorticity Field

We impose:

$$\omega(r,0) = \begin{cases} \Omega\left(1 - \frac{r_0}{r}\right) \text{ si } r < r_0 \\ 0 \text{ si } r \geq r_0 \end{cases} \quad (18)$$

Far from the core we admit that from a certain distance R the vorticity no longer varies along the radial direction. We therefore write

$$r \geq R \; \frac{\partial\omega}{\partial r} = 0 \quad (19)$$

2.5. Finding Stationary Solutions

At very large time scales, if the parameter a is independent of time, we can find an analytical solution of equation (16) because in steady state the partial derivatives with respect to time are identically zero. This allows us to achieve to the following equation:

$$0 = \frac{1}{r}\frac{\partial}{\partial r}\left[a\, r^2\omega + \nu\, r\frac{\partial\omega}{\partial r}\right] \tag{20}$$

$$\left[a\, r^2\omega + \nu\, r\frac{\partial\omega}{\partial r}\right] = \eta \tag{21}$$

As the equation to be integrated depends on the only space variable we can pass from this partial differential equation to an easy to solve first order differential equation.

$$\left[a\, r^2\omega + \nu\, r\frac{d\omega}{dr}\right] = \eta$$

With η a constant.

$$\omega = \omega_0\, e^{-\frac{a}{2\nu}r^2} \tag{22}$$

with

ω_0 the value of the vorticity when equal to zero.

After having found a stationary solution of the equation which characterizes a particular case of the problem, we tried to find a more general solution taking into account this time the spatial and temporal aspects of the problem.

For this, we used a numerical solution method. But before approaching the numerical modeling let us first make the problem dimensionless.

2.6. Adimensionalization of the Vorticity Evolution Equation of the Model

The object of this so-called similarity theory is to reveal dimensionless quantities characteristic of the phenomenon studied. From then on, the only data of a set of these dimensionless numbers can make it possible to define a family of physical situations with the same solution.

Now we will apply this theory to our equation established previously.

To do this, we choose reference quantities:

r_0 is a reference length.

T_0 , Ω designating the reference time

Thus we define the different adimensional quantities of the equation which will be starred:

$$\omega^* = \frac{\omega}{\Omega}$$

$$t^* = \frac{t}{T_0}$$

$$r^* = \frac{r}{r_0}$$

By replacing in the equation

$$\frac{\partial\omega}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[a\, r^2 + \nu\, r\frac{\partial\omega}{\partial r}\right]$$

we obtain the following dimensionless equation.

$$\frac{\partial\omega^*}{\partial t^*} = \frac{1}{r^*}\frac{\partial}{\partial r^*}\left[Ek\frac{\partial}{\partial r^*}(r^*\omega^*) - \left(Ek - Ro.f(t).r^{*2}\right)\omega^*\right] \tag{23}$$

With

$$Ek = \frac{\nu}{\Omega r_0^2} \text{ et } Ro = \frac{a_0}{\Omega}$$

We recall that we set $a = a_0 f(t^*)$

These numbers are called Ekmann's and Rossby's numbers respectively. Their knowledge is fundamental in the study of geophysical movements. The first measures the importance of viscous effects compared to rotational effects while the second compares inertial and rotary effects. In cyclonic flows, the Rossby number is usually very small compared to the Ekmann number. Note that the report

$$\frac{Ro}{Ek} = \frac{a\, r_0^2}{\nu} = Re \tag{24}$$

This ratio is equivalent to a Reynolds number.

The dimensionless initial condition at t*= 0 is

$$\omega^*(r^*, 0) = \begin{cases}(1 - r^*) \text{ si } r^* < 1 \\ 0 \text{ si } r^* \geq 1\end{cases} \tag{25}$$

The dimensionless boundary conditions on the z-axis and very far from the core are

$$\omega^*(0, t^*) = 1 \tag{26}$$

And far from the heart we have

$$r^* \geq R^* \frac{\partial\omega^*}{\partial r^*} = 0 \tag{27}$$

The vorticity evolution equation obtained for the model is of the type of nonlinear partial differential equation impossible to solve analytically. Considering this difficulty of resolution we resorted to a numerical modeling of our problem.

It is a nonlinear partial differential equation that cannot be solved analytically. So to determine the velocity and vorticity fields we will use a numerical resolution model.

2.7. Numerical Modeling of a Cyclone

In this part we will make a model, which with the help of the numerical study allows us to understand the dynamics of a cyclone and the influence of the dimensionless numbers defined previously, therefore the effects of viscosity and convection with respect to the rotational effects. To do this study we have defined a physical domain of small dimensions compared to the phenomenon as a whole. [10]

2.8. Discretization of the Physical Domain

In this paragraph we have assumed a physical domain in the core located between the axis of rotation and the lateral surface of the vorticity tube of radius r_0, which we have divided with a regular discretization step δ between two very close points.

We have considered an elementary segment centered at point P and terminals Wet E (see figure below).

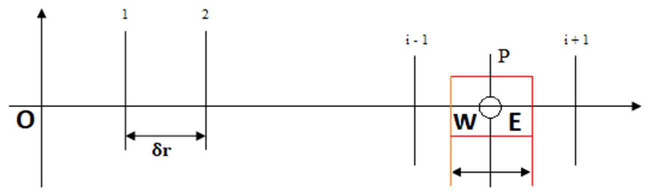


Figure 3. Diagram of discretization of the physical domain.

Let's pose

$r_{i+1}^* = r_i^* + \delta r^*$et $\delta r^* = \frac{1}{im-1}$$i_m$ is the maximum number of nodes.

Integrate our equation (23) between points W and E. We have

By asking $r_t = \frac{\delta t}{\delta r^*}$ then he comes

$$-\left\{\left[\frac{Ek.r_{i-1}^*}{\delta r^*}+\frac{1}{2}[(Ek-Ro.f(t^*)(r_{i-1}^*)^2]\right]\right\}\omega_{i-1}^*+r_i^*\left[2\frac{Ek}{\delta r^*}+\frac{1}{r_t}\right]\omega_i^*$$

$$-\left\{\left[\frac{Ek.r_{i+1}^*}{\delta r^*}-\frac{1}{2}[(Ek-Ro.f(t^*)(r_{i+1}^*)^2]\right]\right\}\omega_{i+1}^*=\frac{r_i^*}{r_t}\omega_i^{*0}$$

With $1 < i < im$

The discretized initial condition is then

$$\omega_1^* = \begin{cases}(1 - r_i^*)\ si\ i < i_0 \\ 0\ si\ r^* \geq i_0\end{cases} \quad (2.2.4.a)$$

Les conditions aux limites sont approchées par

$$\omega_1^* = 0 \text{ and } \omega_{im-1}^* - \omega_{im}^* = 0 \qquad (28)$$

This system of trigonal equations is solved thanks to the algorithm of Thomas. [11, 12]

3. Results

3.1. Influence of the Decrease in Aspiration Speed over Time

In this part we posed the radial dimensionless suction velocity is given by

$$\vartheta_r^* = -(1-e^{t^*}).r^*$$

Figures 4 and 5 show the profiles of the vorticity and the ortho-radial velocity as a function of the radial coordinate for an Ekman number equal to 1 and a low Rossby number. As time increases, the radius of the vortex core shrinks due to the decrease in the intensity of the suction velocity. To compensate for this narrowing, the diffusion effects are amplified which results in a spreading of the ortho-radial velocity. [13]

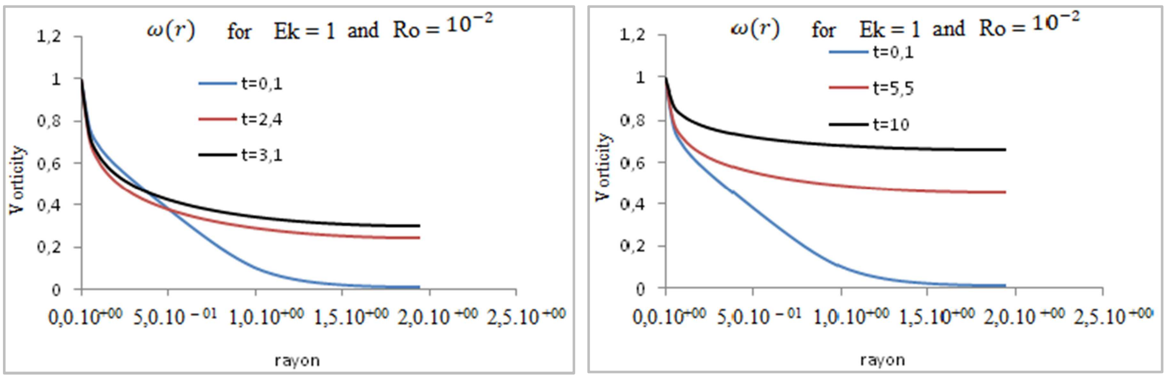


Figure 4. Variation of vorticity as a function of radius at different times Ek = 1; Ro= 10^{-2}.

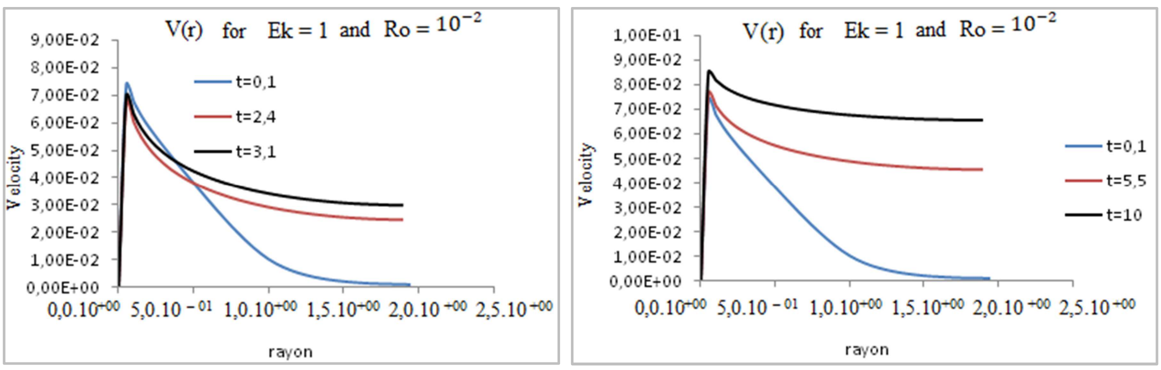
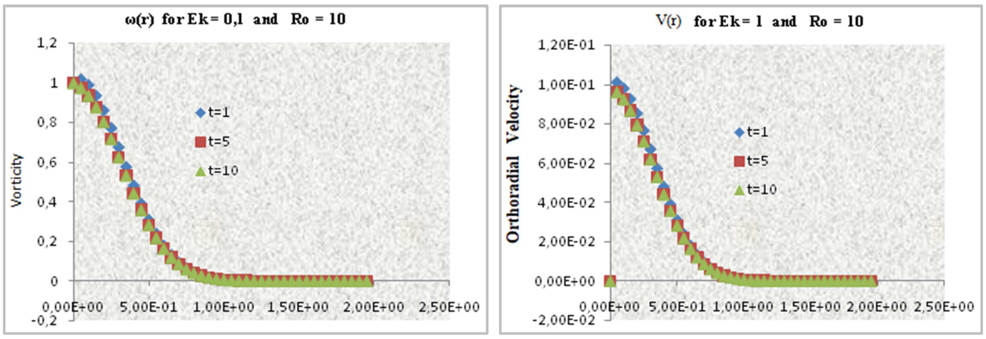


Figure 5. Variation of velocity as a function of radius at different times Ek = 1; Ro = 10^{-2}.

When the Rossby number increases up to the value 10, the steady state is quickly reached if the suction speed does not vary over time as we can see in Figures 4. On the other hand, if the latter decreases over time we obtain the same phenomena as before (see figures 4) In other words the Rossby number is not large enough to disturb the topology of the flow. The effect of the Ekman number is still preponderant.



a b

Figure 6. *a) Variation of vorticity as a function of the radial abscissa at different times Ek = 1; Ro= 10. b) Variation of the ortho-radial component as a function of the radial abscissa at different times Ek = 1; Ro= 10.*

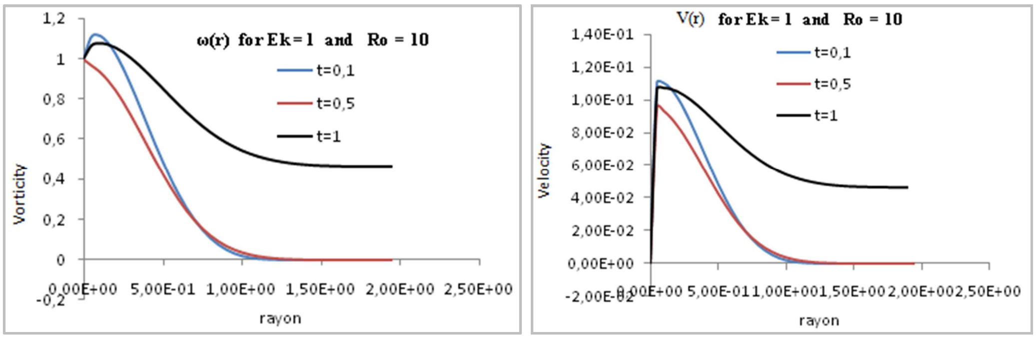


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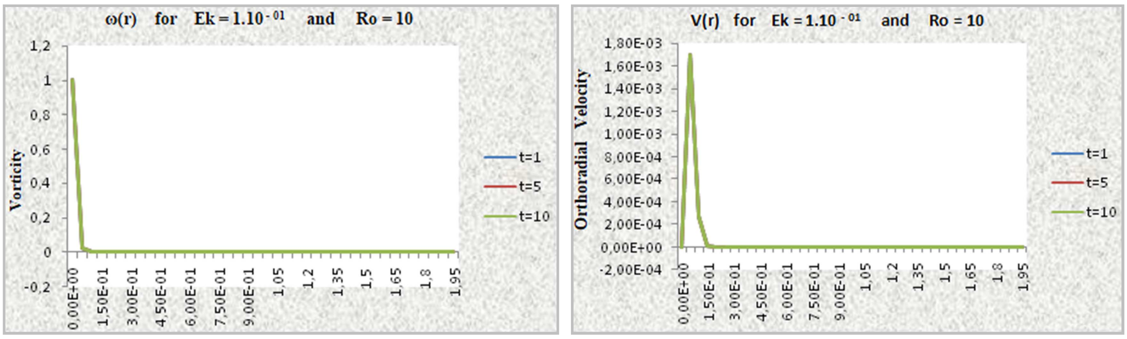
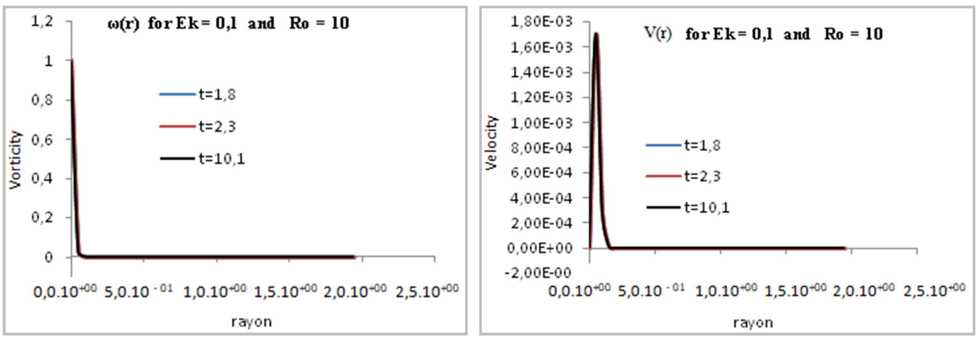
Figure 7. *a) Variation of vorticity as a function of radius at different times Ek = 1; Ro= 10. b) Variation of velocity as a function of radius at different times Ek = 1; Ro= 10.*



a b

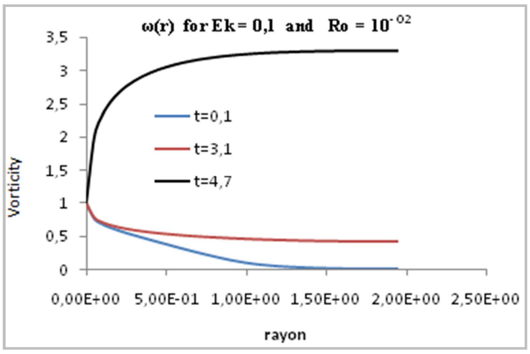
Figure 8. *a) Variation of vorticity as a function of the radial abscissa at different times Ek = 0,1; Ro = 10. b) Variation of the ortho-radial component as a function of the radial abscissa at different times. Ek = 0, 1; Ro = 10.*



a b

Figure 9. *a) Variation of vorticity as a function of radius at different times Ek = 0,1; Ro= 10. b) Variation of velocity as a function of radius at different times Ek = 0,1; Ro= 10.*

Figures 5 and 6 show that when the Ekman number is low (Ek = 0.1) the attenuation of the ortho-radial velocity no longer has any effect on the profiles of vorticity and the ortho-radial velocity. [13, 14]



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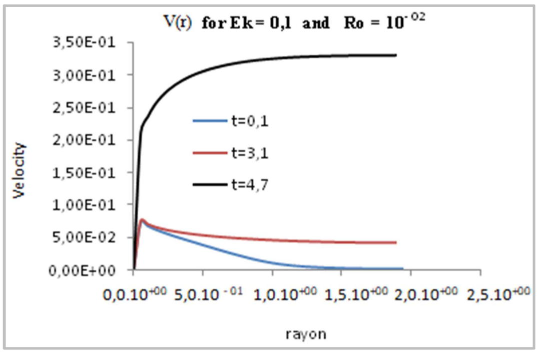

b

Figure 10. *a) Variation of vorticity as a function of radius at different times k = 1; Ro=* 10^{-2}. *b) Variation of velocity as a function of radius at different times Ek = 1; Ro= 10-².*

3.2. Case Where the Aspiration Speed Increases Over Time

Now we consider the case where the suction velocity is an increasing function of time is given by

$$\vartheta_r^* = -\left(1 - e^{-t^*}\right).r^*$$

When the intensity of the suction increases the vortex regime is supercharged. The radius of the core of the vortex as well as its intensity increase rapidly. The viscous effects are quickly engulfed by the inertial effects which will lead to an early turbulent regime.

We looked at the effects of the variation of the radial velocity as a function of time. It appears from the results obtained by the numerical simulations that the Ekman number is the most important parameter in the dynamics of the cyclone. Its increase results in a growth of the rotational movement to the detriment of the effects of viscosity.

4. Conclusion

The study presented in this article is a study of the dynamic analysis of cyclone phenomena.

With regard to the modeling and the theoretical study, we note first of all that the conditions initially imposed have been respected. We have treated the case of a supposedly vertical and isothermal cyclone with simple axis symmetry and hydrodynamic assumptions in order to determine the longitudinal and radial components of the velocity field. The general Navier–Stokes evolution equation is put in a rather simple form but impossible to solve analytically due to the presence of non-linear terms.

The equations of the continuous domain are then solved by the finite volume control method.

The results of the analysis of the fields of vorticity and speed conditioned by the adimensional numbers of Rossby and Ekmann very characteristic in dynamics of the fluids as well as the numerical parameters also brought out the contribution of the method of finite volumes of control on the study of the dynamics our problem. Indeed, the examination of the properties of speed and vorticity made it possible to illustrate the distinction between the amplification and the attenuation of the vorticity therefore of the cyclone.

As a result, the conclusions obtained will be more accurate and relevant because the model reflects reality more faithfully. The idea of making the dynamic study of a cyclone model with the explicit influence of the radial velocity to our knowledge inaccessible until now, remains attractive. We faced a lack of data, the cyclone is a natural phenomenon whose study is a little less advanced. However, we can consider that our analysis of the influence of the radial velocity on the dynamics of the tropical cyclone can be the starting point in a more general physical framework.

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